

Module 22 : Non-linear fiber optics I

Lecture : Non-linear fiber optics I

Objectives

In this lecture you will learn the following

- Basics of Non-Linear Fiber Optics
- Kerr Non-linearity
- Mixing of frequencies
- Wave Equation for Non-linear Medium
- Non-Linear Schrodinger Equation
- Qualitative picture of wave propagation in a Non-linear medium

Basics of Non-Linear Fiber Optics

What is Non-linear fiber optics?

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When the optical intensity inside an optical fiber increases, the refractive index of the fiber gets modified. The wave propagation characteristics become a function of optical power. Unlike linear fiber optics, where the propagation constant is a function of fiber and the wavelength only, the propagation constant becomes a function of optical power in addition to the other parameters.

Inside a single mode optical fiber, an optical power of few tens of mW may drive the medium into non-linearity.

Why should we Study Non-linear Fiber optics?

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When the optical fiber becomes non-linear the pulse propagation gets significantly modified.

Also new frequencies get generated inside the optical fiber. The spectrum of the output

signal is not same as that of the input signal.

To observe the non-linearity in the bulk medium generally a large optical power is required. Whereas, optical non-linearity can be easily observed inside an optical fiber with very low optical power of few tens of mW. This can be explained as follows:

The efficiency of non-linear process is proportional to the optical intensity (Optical power per unit area) and the interaction length.

For a bulk medium the efficiency is

$$\eta_{bulk} = P / \lambda$$

Where P is the optical power and λ is the optical wavelength.

On the other hand the efficiency for the optical fiber is

$$\eta_{fiber} = \frac{P}{\pi a^2 \alpha}$$

Where a is the core radius of the fiber and α is the attenuation constant of the fiber. Note that $P / \pi a^2$ is the power density in the fiber core, and $1 / \alpha$ is the effective interaction length on the fiber. Since α is very small for the optical fiber, the interaction length is few Km of a typical single mode optical fiber. Also since the core radius is very small, few micrometer, the optical intensity inside the core is very high.

The ratio of the fiber to bulk efficiency (assuming $a = 4\mu\text{m}$, $\alpha = 0.3\text{dB/km} = 3.456 \times 10^{-5} \text{ nepers/m}$, $\lambda = 1550\text{nm}$) is

$$\frac{\eta_{fiber}}{\eta_{bulk}} = \frac{\lambda}{\pi a^2 \alpha} \approx 10^9$$

That means the non-linear efficiency inside a fiber is about billion times more than that in the bulk medium.

It is therefore easy to get non-linearity inside an optical fiber than in a bulk medium.

Since the non-linearity affects the signal propagation as a whole, there may be two approaches to handle it.

- Avoid non-linearity in the system by keeping low optical power levels.
- Understand the non-linear signal propagation and make intelligent use of it for increasing the transmission capability of the fiber.

The first option does not seem very feasible since the optical power has to be increased for long distance communication. Also in a multi-channel system like WDM, even if each channel has low power, the total power of all channels together is large enough to drive the optical fiber into non-linear regime.

The second option therefore is more appropriate and desirable. This option is interesting and at the same time very challenging because the non-linear pulse propagation is a very complex phenomenon. It requires far more in-depth understanding of the wave

propagation. Nevertheless we will investigate some of the aspects of the non-linear pulse propagation on an optical fiber. The saving grace is, the non-linear effects are weak and therefore certain approximations can be rightly made in the analysis.

Kerr Non-linearity

In the presence of intense optical field, the induced electric polarization inside the material can be generally written as

$$\mathbf{P} = \epsilon_0 \left\{ \chi^{(1)} \cdot \mathbf{E} + \chi^{(2)} : \mathbf{E}\mathbf{E} + \chi^{(3)} \vdash \mathbf{E}\mathbf{E}\mathbf{E} + \dots \right\}$$

Where ϵ_0 is the free-space permittivity, and $\chi^{(1)}$, $\chi^{(2)}$, $\chi^{(3)}$ are the first, second, and third order susceptibilities of the medium. The susceptibilities are in general tensors.

Quantity $1 + \chi^{(1)}$ essentially is the dielectric constant of the medium. The first order susceptibility represents linear property of the medium.

The higher order susceptibilities give non-linear effects. These susceptibilities depend upon the molecular structure of the material. For Pure silica glass, the $\chi^{(2)}$ is very small.

The lowest order non-linear susceptibility which affects the optical fibers is the third order susceptibility $\chi^{(3)}$. This susceptibility can provide many effects like, third harmonic generation, Four Wave mixing (FWM), and non-linear refraction.

Let investigate here the phenomenon of non-linear refraction.

Due to third order susceptibility, the refractive index of the medium can be written as

$$n = n_0 + n_2 |E|^2$$

For an electromagnetic wave the Poynting vector (power density) is proportional to the square of the magnitude of the electric field. The refractive index of the medium has two components, n_0 , the linear refractive index which could be a function of frequency, and $n_2 |E|^2$, the non-linear refractive index which is a function of optical power density.

This non-linearity is called the **Kerr Non-linearity** and the effect is called the **Kerr-effect**.

The non-linear index coefficient n_2 is related to the third order susceptibility through a relation

$$n_2 = \frac{3}{8n_0} \chi^{(3)}$$

For glass $n_2 = 3.2 \times 10^{-12} \text{ m}^2/\text{W}$.

Mixing of frequencies

One of the important feature of medium non-linearity is generation of new frequencies. This can be explained as follows.

Let there are three electric fields E_1, E_2, E_3 with three frequencies $\omega_1, \omega_2, \omega_3$ and photon momenta $\kappa_1, \kappa_2, \kappa_3$ respectively incident on a medium with high third order susceptibility.

The non-linear polarization in presence of these fields is

$$\mathbf{P}_{NL} = \epsilon_0 \chi^{(3)} \mathbf{E}_1 \mathbf{E}_2 \mathbf{E}_3$$

The interaction of three photons takes place inside the material and a new photon is created such that the total energy and momentum is conserved. If the fourth photon has a frequency ω_4 and momentum κ_4 , then we must have

$$\omega_4 = \omega_1 \pm \omega_2 \pm \omega_3$$

$$\kappa_4 = \kappa_1 \pm \kappa_2 \pm \kappa_3$$

This process is called **Four Wave Mixing (FWM)** or **Four Photon Mixing**.

The conservation of momentum requires what is called the phase matching of the signals inside the medium.

One of the combinations of the frequencies which give the new frequency close to the original frequencies is

$$\omega_4 = \omega_1 + \omega_2 - \omega_3$$

There are 9 combinations possible in general for the new frequencies plus three combinations which would produce new frequencies same as the original frequencies.

They are

$$\omega_1, \omega_2, \omega_3$$

$$\omega_1 + \omega_2 - \omega_3, \omega_1 + \omega_3 - \omega_2, \omega_3 + \omega_2 - \omega_1,$$

$$2\omega_1 - \omega_2, 2\omega_1 - \omega_3, 2\omega_2 - \omega_1, 2\omega_2 - \omega_3, 2\omega_3 - \omega_1, 2\omega_3 - \omega_2$$

However, if the two input frequencies are same then we get limited frequencies.

If $\omega_1 = \omega_2$, then the new frequency is

$$\omega_4 = 2\omega_1 - \omega_3$$

And if $\omega_2 = \omega_3$, then there is no new frequency generated since

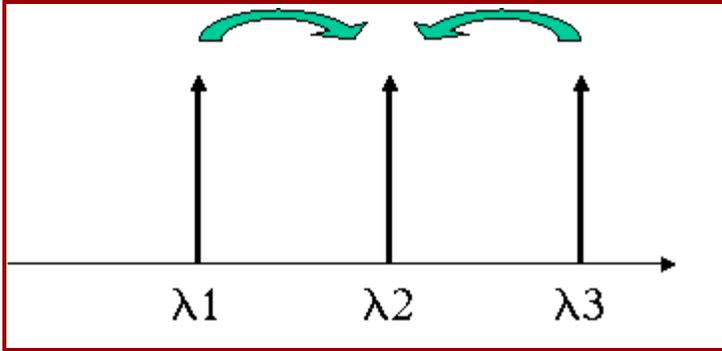
$$\omega_4 = \omega_1$$

In a WDM system when the channels are equi-spaced, $\omega_2 = \omega_1 + 2\Delta\omega$, $\omega_3 = \omega_1 + \Delta\omega$

And the new frequency is

$$\omega_4 = \omega_1 + \omega_2 - \omega_3 = \omega_1 + \omega_1 + 2\Delta\omega - (\omega_1 + \Delta\omega) = \omega_3$$

This then means that the two neighboring channels of a WDM system exchange power with the middle channel. Or in other words there is a cross talk introduced between the WDM channels in the presence of four wave mixing.



The efficiency of the FWM depends upon the phase matching between the waves. Since due to dispersion on an optical fiber, different frequencies travel with different speeds, there is phase mismatch between the channels.

The FWM efficiency is given as

$$\eta = \frac{\alpha^2}{\alpha^2 + \Delta\beta^2} \left\{ 1 + \frac{4e^{-\alpha L} \sin^2(\Delta\beta L/2)}{(1 - e^{-\alpha L})^2} \right\}$$

Where $\Delta\beta$ is the difference in the propagation constants of the various waves due to dispersion.

$$\begin{aligned} \Delta\beta &= \beta_1 + \beta_2 - \beta_3 - \beta_4 \\ &= \frac{2\pi\lambda^2}{c} (f_1 - f_3)(f_2 - f_3) \left[D(\lambda) - \frac{\lambda^2}{c} \left(\frac{f_1 - f_2}{2} - f \right) \frac{dD}{d\lambda} \right] \end{aligned}$$

And $\beta_1, \beta_2, \beta_3, \beta_4$ are the phase constants and f_1, f_2, f_3, f_4 are the frequencies of the four waves inside the optical fiber. D is the dispersion at the wavelength of interest.

For equi-spaced channels $|f_1 - f_3| = |f_2 - f_3| = \Delta f$, and we get the phase mismatch as

$$\Delta\beta = \frac{2\pi\lambda^2}{c} \Delta f^2 \left[D(\lambda) - \frac{\lambda^2}{c} \Delta f \frac{dD}{d\lambda} \right]$$

For generation of FWM components, the dispersion and its slope should be as small as possible. In this situation a dispersion flattened fiber is to be used.

However, in a DWDM system where the FWM is undesirable (since FWM introduces cross-talk between different channels), the fiber should have some dispersion. A low but non-zero dispersion fiber is necessary for WDM transmission for suppressing FWM.

- **Wave Equation for Non-linear Medium**

In presence of non-linearity the refractive index is intensity dependent. So if the dielectric constant is appropriately replaced with non-linear term we get the wave equation for the non-linear medium.

The wave equation in general then is given as

$$\nabla^2 E = \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} + \chi^{(1)} \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} + \chi^{(2)} \frac{1}{c^2} \frac{\partial^2 EE}{\partial t^2} + \chi^{(3)} \frac{1}{c^2} \frac{\partial^2 EEE}{\partial t^2} + ..$$

The term $(1 + \chi^{(1)})$ is the dielectric constant of the medium. The linear refractive index of the medium is $n = \sqrt{1 + \chi^{(1)}}$. The last two terms in the equation are the non-linear terms.

For silica glass since $\chi^{(2)}$ is negligibly small, the wave equation reduces to

$$\nabla^2 E = \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} + \chi^{(1)} \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} + \chi^{(3)} \frac{1}{c^2} \frac{\partial^2 EEE}{\partial t^2} + ..$$

For time harmonic field with angular frequency ω , the equation reduces to

$$\nabla^2 E + \frac{\omega^2}{c^2} (n + n_2 |E|^2) E = 0$$

Where the non-linear coefficient has been defined above.

The equation is not very to solve. However, under the assumption that the electric field evolves slowly due to non-linear effects, the equation can be linearized.

- **Non-Linear Schrodinger Equation**

The non-linear Schrodinger equation (NSE) has special importance in the propagation of light in the optical fibers in presence of non-linearity.

Although the equation has been derived from the classical Maxwell's equations, the name has been assigned to the equation due to the fact that in presence of non-linearity, the energy packets show particle kind of behavior (this aspect will become clear when we discuss solitonic propagation).

To derive the NSE from the wave equation some assumptions have to be made which are following.

- The fiber is a single mode fiber.
- All scattering effects in the fiber are neglected.
- The amplitude envelop of the wave changes slowly with respect to the carrier. That is, the electric field is quasi-monochromatic meaning fractional bandwidth of the wave is very small. This assumption is quite valid for most of the communication signals, since the carrier frequency is of the order of 10^{14} Hz (optical frequency) and the modulation frequency (same as data rate) is of the order of $10^9 - 10^{10}$ Hz.
- The non-linearity is a small perturbation to the linear term.
- Polarization of the wave remains same throughout the propagation.
- The transverse distribution of the modal fields is unaffected by the non-linearity.

The electric field can be written as

$$\mathbf{E} = \mathbf{F}(\mathbf{r}, \phi) A(z) e^{j(\omega t - \bar{\beta} z)}$$

Where the cylindrical coordinate system is assumed and $F(r, \phi)$ is the modal field distribution and $A(z)$ is a slowly varying envelope function of z .

The modal field distribution satisfies the linear wave equation giving

$$\nabla^2 F + \left(\frac{\omega^2 n^2}{c^2} - \bar{\beta}^2 \right) F = \nabla^2 F + (\beta_0^2 - \bar{\beta}^2) F = 0$$

Where β_0 is the phase constant of the carrier in the optical fiber and $\bar{\beta}$ is the modal propagation constant. This equation is identical to the one solved in the module on wave propagation inside an optical fiber.

Assuming that the envelop function varies slowly as a function of distance, the terms like $\partial^2 A / \partial z^2$ are neglected and then the envelop function satisfies the equation

$$2j\beta_0 \frac{\partial A}{\partial z} + (\beta_0^2 - \bar{\beta}^2) A = 0$$

The propagation constant $\bar{\beta}$ gets contributions from two effects, one due to dispersion, that is its dependence on frequency, and other due to loss and non-linear effects. Let us therefore write

$$\bar{\beta} = \beta(\omega) + \Delta\beta$$

Where

$$\Delta\beta = \frac{\omega_0}{c} \frac{\int_0^{2\pi} \int_0^{\infty} \Delta n |F(r, \phi)|^2 r dr d\phi}{\int_0^{2\pi} \int_0^{\infty} |F(r, \phi)|^2 r dr d\phi}$$

And

$$\Delta n = n_2 |E|^2 - j \frac{\alpha}{2} \frac{c}{\omega_0}$$

α is the attenuation constant of the fiber.

Now since $(\bar{\beta} - \beta_0) \ll \beta_0$, we can approximate, $(\beta_0^2 - \bar{\beta}^2) \approx 2\beta_0(\beta_0 - \bar{\beta})$ and the equation of the envelop function becomes

$$\frac{\partial A}{\partial z} - j(\beta_0 - \beta(\omega) - \Delta\beta)A = 0$$

Let us take the Fourier transform of this equation over time to get

$$\frac{\partial \tilde{A}}{\partial z} - j(\beta_0 - \beta(\omega) - \Delta\beta)\tilde{A} = 0$$

Where \tilde{A} is the Fourier transform of A defined as

$$\tilde{A}(\omega - \omega_0) = \int_{-\infty}^{\infty} A(t) e^{-j(\omega - \omega_0)t} dt$$

And the inverse Fourier transform is defined as

$$A(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{A}(\omega - \omega_0) e^{j(\omega - \omega_0)t} d\omega$$

Now let us expand $\beta(\omega)$ in Taylor series around β_0 as

$$\beta(\omega) = \beta_0 + (\omega - \omega_0)\beta_1 + \frac{1}{2}(\omega - \omega_0)^2 \beta_2 + \frac{1}{6}(\omega - \omega_0)^3 \beta_3 + \dots$$

Where,

$$\beta_n = \left(\frac{\partial^n \beta}{\partial \omega^n} \right)_{\omega=\omega_0}$$

Substituting for $\beta(\omega)$ in the envelop equation and retaining only up to the second derivative terms of β , we get

$$\frac{\partial \tilde{A}}{\partial z} + j(\omega - \omega_0)\beta_1 \tilde{A} + j \frac{1}{2}(\omega - \omega_0)^2 \beta_2 \tilde{A} + j\Delta\beta \tilde{A} = 0$$

Taking inverse Fourier transform the equation of the envelop is

$$\frac{\partial A}{\partial z} + \beta_1 \frac{\partial A}{\partial t} - j \frac{1}{2} \beta_2 \frac{\partial^2 A}{\partial t^2} + j \Delta\beta A = 0$$

Substituting for $\Delta\beta$ in the equation the **Non-Linear Schrodinger Equation (NSE)** as

$$\frac{\partial A}{\partial z} + \beta_1 \frac{\partial A}{\partial t} - j \frac{1}{2} \beta_2 \frac{\partial^2 A}{\partial t^2} + \frac{\alpha}{2} A + j\gamma |A|^2 A = 0$$

Where we have defined the non-linearity coefficient γ as

$$\gamma = \frac{n_2 \omega_0}{c A_{eff}}$$

The parameter A_{eff} is called the effective core area and is given by

$$A_{eff} = \frac{\left(\int_0^{2\pi} \int_0^{\infty} |F(r, \phi)|^2 r dr d\phi \right)^2}{\int_0^{2\pi} \int_0^{\infty} |F(r, \phi)|^4 r dr d\phi}$$

We can identify the various terms of the NSE as follows:

- The first term gives the rate of change of the wave envelop as a function of distance.
- The second term is related to the group velocity since $\beta_1 = (\partial\omega / \partial\beta)^{-1} = (\text{Group velocity})^{-1} = \text{Group delay}$
- The third term gives the group velocity dispersion (GVD) of the envelop.
- The fourth term is due to the loss (attenuation) on the optical fiber.
- The fifth term is due to the fiber Kerr non-linearity.

In subsequent module we will solve the NSE to see the effect of each term on pulse propagation in an optical fiber.

By putting $\gamma = 0$ in the NSE we can get the pulse propagation equation for the linear medium.

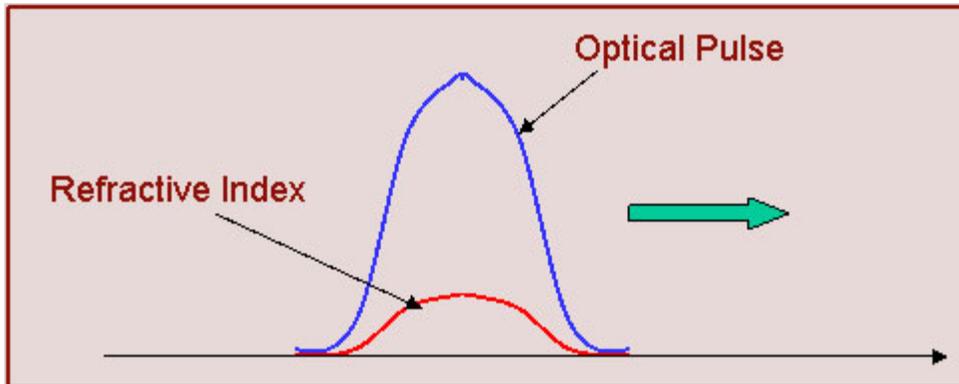
• Qualitative picture of wave propagation in a Non-linear medium

We have seen earlier that the light is guide in the core of the optical fiber, because

refractive index of the core is higher compared to its surrounding medium, the cladding.

In other words the energy has tendency to concentrate in the region of higher refractive index.

Let us now consider an optical pulse (say of Gaussian shape). Due to Kerr non-linearity the entire pulse does not see the same refractive index, but the refractive index is maximum at the peak of the pulse and minimum at the edges of the pulse as shown in Fig.



The refractive index profile along the length of the fiber is no more constant now but has a shape same as the pulse power profile. Since the refractive index profile is created by the pulse itself, it slides along with the pulse.

The pulse energy has now preference to stay near its peak, as refractive index is higher here. The non-linearity has tendency to compress the pulse.

If there is a dispersion in the medium, the pulse energy has tendency to spread away as it travels along the fiber. In a linear medium therefore the pulse broadens.

The non-linearity has effect opposite of the dispersion. It may then be used to counteract the dispersion on the fiber.

If the compressing effect due to non-linearity is completely balanced by the spreading effect due to the dispersion, then the pulse can transmit indefinitely without either expansion or contraction. The pulse will therefore travel undistorted on the optical fiber for infinite length.

From communication view point this aspect is very interesting, since a high data rate can be sent on the long length of the fiber with out worrying about the dispersion.

It can very easily be noted that at any point in time, the non-linear effect should not become weaker compared to the dispersion. Because, if dispersion overcomes the non-linearity, the pulse spreads bring the peak of the pulse down. This in turn reduces the non-linear effect, making the dispersion to further dominate and broaden the pulse further. The process is regenerative and the non-linearity can never catch up with the dispersion if it loses once. Every effort therefore has to be made to maintain the power level of the pulse.

In the presence of fiber loss, the pulse power decreases with distance making non-linearity weaker. Appropriate amplification mechanisms have to be worked out for maintaining the optical power in the pulse.

Recap

In this lecture you have learnt the following

- **Basics of Non-Linear Fiber Optics**
- Kerr Non-linearity
- **Mixing of frequencies**
- **Wave Equation for Non-linear Medium**
- **Non-Linear Schrodinger Equation**
- **Qualitative picture of wave propagation in a Non-linear medium**

Congratulations, you have finished Lecture 22. To view the next lecture select it from the left hand side menu of the page